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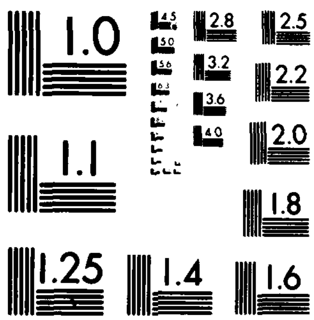

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A RAPID AND ACCURATE SOLUTION FOR THE POISSON EQUATION WITH NEUMANN BOUNDARY CONDITIONS ON GENERAL DOMAINS

S. Abdallah

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17 December 1981
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This memorandum presents a rapid and accurate solution for the Poisson equation with Neumann boundary conditions on general domains. The stream-like function formulation is used to obtain two modified Poisson equations with Dirichlet boundary conditions. The first equation is solved for the flux components of the dependent variable, and are used in calculating Dirichlet boundary conditions for the second Poisson equation. Two integral constraints are to be satisfied in the present formulation. The first constraint is a consequence of Green's first integral theorem and the second constraint is a result of the present

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20. analysis. A new procedure is implemented in the boundary conditions to satisfy the second integral constraint. This technique is applied to calculate the static pressure from a Poisson equation with Neumann boundary conditions in the solution of Navier-Stokes equations (velocity pressure formulation). Numerical results for the incompressible viscous flow in a driven cavity are presented and compared with the available numerical results in the literature.

Subject: A Rapid and Accurate Solution for the Poisson Equation With Neumann Boundary Conditions on General Domains

References: See page 12

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Table of Contents

List of Figures

17 December 1981
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Nomenclature

A	area of the solution domain
C	contour, enclosing the area, A
C_1, C_2	arbitrary constants
D	parameter defined in Eq. (17)
dC	incremental distance along contour, C
f	Neumann boundary conditions
n	outward normal to the boundaries of the solution domain
P	dependent variable and static pressure
Re	Reynolds number
R, Q	flux components defined in Eqs. (4) and (5)
u	velocity component in the x-direction
v	velocity component in the y-direction
x_1, s_1	arbitrary points on the integration reference line
σ	source term, Eqs. (1) and (17)
χ	streamlike function

INTRODUCTION

A rapid and accurate solution to the discrete Poisson equation is essential for the computational fluid dynamic applications and other fields [1-8].* In recent years, highly efficient direct solutions for the Poisson equation have been developed using cyclic reduction methods, tensor product methods, the Fourier series method, and a few others [9-13]. All these methods have one or more limitations [1].

The existence and uniqueness of a solution to the Poisson equation with Neumann (derivative) boundary conditions requires an extra constraint which relates the source term and the Neumann conditions. This condition is not exactly satisfied in the numerical solutions due to truncation errors which lead to slow convergence. The slow convergence of the solution for the Poisson equation has restricted many of the numerical solutions [1]. It is worth mentioning here that two methods to improve the convergence of the numerical techniques are given in References [2] and [3]. Briley [2] modified the source term in the Poisson equation to match the boundary conditions, while Miyakoda [3] modified the boundary conditions instead.

This memorandum presents a new method for solving the Poisson equation with Neumann boundary conditions on general domains. The streamlike function formulation [15] is used to transform the Poisson equation with Neumann boundary conditions into two Poisson equations with Dirichlet boundary conditions. These two Poisson equations can be solved numerically using Iterative and Direct techniques [9-14]. The Direct solvers are applicable in the present case because of the resulting Dirichlet boundary conditions.

This technique is applied to calculate the static pressure from a Poisson equation with Neumann boundary conditions in the solution of Navier-Stokes equations (velocity pressure formulation). Numerical results are obtained for the incompressible viscous flow in a driven cavity. The momentum equations are integrated for the velocity components using an explicit marching technique, while the two Poisson equations for the pressure are solved using the successive over-relaxation method. The computed results are presented and compared with the numerical results of References [16-20].

MATHEMATICAL FORMULATION

The two-dimensional Poisson equation with Neumann (derivative) boundary conditions can be written in the following form:**

*Numbers in brackets [ ] designate References, Page 12.
**This analysis is performed for the two-dimensional Poisson equation, but it is directly valid for the axisymmetric case.

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = \sigma(x,y) \tag{1}$$

and

$$\frac{\partial P}{\partial n} = f(C) \tag{2}$$

where P is the dependent variable, σ is the source term, n is the outward normal to the boundaries of the solution domain, and f is the derivative of the dependent variable specified along the boundary contour C.

The solution to Eq. (1) with boundary conditions, Eq. (2), exists if the following integral constraint which results from Green's first integral theorem is satisfied:

$$\int\int \sigma(x,y)\ dA = \oint f(C)\ dC \tag{3}$$

where A is the area of the solution domain and dC is the incremental distance along C.

The solution to Eq. (1) is obtained using the streamlike function formulation [15] as follows:

Let

$$R = \frac{\partial P}{\partial x} \tag{4}$$

and

$$Q = \frac{\partial P}{\partial y} \tag{5}$$

Using the above equations, Eq. (1) can be written in the following form:

$$\frac{\partial R}{\partial x} + \frac{\partial Q}{\partial y} = \sigma(x,y) \tag{6}$$

$$\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial x} = 0 \quad . \tag{7}$$

Equations (6) and (7) are solved for the flux components R and Q using the streamlike function formulation [15]. These two components are expressed in terms of the streamlike function, χ, and the source term, σ, in Eq. (6) as:

$$R = \frac{\partial \chi}{\partial y} + \int_{x_1}^{x} \sigma(x,y)\, dx \tag{8}$$

and

$$Q = -\frac{\partial \chi}{\partial x} \quad . \tag{9}$$

Equations (8) and (9) satisfy Eq. (6) identically and transform Eq. (7) into the following second-order differential equation in χ:

$$\frac{\partial^2 \chi}{\partial x^2} + \frac{\partial^2 \chi}{\partial y^2} = -\frac{\partial}{\partial y}\int_{x_1}^{x} \sigma(x,y)\, dx \quad . \tag{10}$$

The boundary conditions for Eq. (10) in terms of the streamlike function, χ, are obtained from the Neumann conditions for P given by Eq. (2).

$$\chi = \int_{s_1} f(C)\, dC - \int\int_{x_1}^{x} \sigma(x,y)\, dx\, dy + C_1 \tag{11}$$

where x_1 and s_1 are arbitrary points on the integration reference line and C_1 is an arbitrary constant.

From the solution of Eq. (10) with the boundary conditions, Eq. (11), the streamlike function is known everywhere in the solution domain. The flux components R and Q are calculated from the streamlike function, χ, using Eqs. (8) and (9). The integration of these flux components along the boundaries will result in the following Dirichlet boundary condition for P:

$$P = \int_{x_1}^{x} \frac{\partial \chi}{\partial y}\, dx + \int_{x_1}^{x}\int_{x_1}^{x} \sigma(x,y)\, dx\, dx - \int \frac{\partial \chi}{\partial x}\, dy + C_2 \tag{12}$$

where C_2 is an arbitrary constant.

The solution of Eq. (1) with the Dirichlet boundary conditions, Eq. (12), for the dependent variable, χ, closes the problem.

Integral Constraints

Two integral constraints have to be satisfied in the numerical solution of Eqs. (1) and (10). These constraints result from the integration of Eqs. (6) and (7) over the area of the solution domain. The first integral constraint is the existence condition given by Eq. (3). This condition exists in the Dirichlet boundary conditions when the integrals in Eq. (11) are evaluated over the total area and the closed contour, C. The presence of the integral constraint, Eq. (3), in the boundary conditions, Eq. (11), causes a discontinuity in the streamlike function, χ, at one point on the boundary if the constraint, Eq. (3), is not exactly satisfied. This discontinuity is treated in a manner equivalent to the treatment of the existence condition in Reference [2]. The area average of the error between the two sides of Eq. (3) is subtracted uniformly from the source term of Eq. (1).

The second integral constraint for the solution of Eq. (1) is

$$\oint \frac{\partial P}{\partial C}\, dC = 0 \quad . \tag{13}$$

A similar technique to the treatment of the first integral constraint, Eq. (3), was tried to satisfy Eq. (13), but without success. In the present study, Eq. (13) is satisfied by using Neumann conditions for P on one portion of the contour, C. This procedure is simple and showed excellent results.

APPLICATION

The incompressible viscous flow in a driven cavity, Figure 1, is chosen as a test problem in the present study because of its simple geometry and the available numerical results for comparison. In the velocity pressure formulation of the Navier-Stokes equations, the pressure is recovered from a Poisson equation with Neumann boundary conditions. In this equation, conservation of mass is enforced.

Governing Equations

The two-dimensional Navier-Stokes equations are given below in nondimensional form. The velocity of the moving boundary and the cavity width are used as the characteristic parameters.

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = -\,\frac{\partial P}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{14}$$

$$\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} = -\,\frac{\partial P}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{15}$$

and the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{16}$$

where u is the velocity component in the x-direction, v is the velocity component in the y-direction, P is the static pressure, and Re is the Reynolds number.

The following Poisson equation for the pressure is obtained by adding the derivatives of Eqs. (14) and (15), with respect to x and y, respectively:

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{y^2} = \sigma(x,y) \tag{17a}$$

where

$$\sigma(x,y) = -\,\frac{\partial D}{\partial t} - \frac{\partial}{\partial x}\left(u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}\right)$$

$$-\,\frac{\partial}{\partial y}\left(u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}\right) + \frac{1}{Re}\left(\frac{\partial^2 D}{\partial x^2} + \frac{\partial^2 D}{\partial y^2}\right) \tag{17b}$$

and

$$D \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad . \tag{17c}$$

The continuity equation, Eq. (16), is enforced in the first term on the right-hand side of Eq. (17).

Equations (14) and (15) are solved for the velocity components u and v, respectively, while Eq. (17) is solved for the pressure, P. Boundary conditions for the above equations in terms of the primitive variables are shown in Figure 2 and Dirichlet boundary conditions for the streamlike function, χ, and the pressure, P, are shown in Figure 3. In this case, the integration reference line is chosen parallel to the y-axis and passing through the center of the lower wall, Figure 3. The Neumann conditions for the pressure are retained on the upper moving wall for two reasons; the first is to satisfy the integral constraint, Eq. (13); and the second reason is to eliminate the effect of the singularities at the upper corners on the solution of the second Poisson equation.

RESULTS

A numerical code is written in FORTRAN IV for the solution of the governing Eqs. (14), (15), and (17). The momentum Eqs. (14) and (15) are integrated for the velocity components u and v, respectively, using an explicit marching technique [1]. The static pressure is obtained from the solution of Eq. (17) using the present method. The numerical solutions of the Poisson equations are calculated using the Successive Over-Relaxation method (SOR).

The numerical code for the Poisson equation has been tested against analytical solutions. Secondly, the numerical solutions of a fully-developed flow in horizontal and vertical channels have been used to test the combination of the marching explicit solution and Poisson solver.

Figures 4 and 5 show the stream function and the static pressure contours for $R_e = 1$. The streamlines in Figure 4 are symmetric about the cavity vertical centerline, as expected, and are plotted here for a checkup. The static pressure contours shown in Figure 5 are almost antisymmetric about the cavity vertical centerline. In this case, the static pressure is set equal to zero at the center of the lower wall.

Two groups of results are found in the literature for the cavity problem and are used for comparison in the present study. The first set of results is obtained using finite element techniques and the second set of results is obtained using finite difference methods. Figure 6 shows the velocity component u for $R_e = 10$ on the horizontal line y = 1/14 from the moving wall. It can be seen from Figure 6 that the present results are in good agreement with the results of the finite elements [19-20] and the finite difference methods [16-18]. Figures 7 8, and 9 show the velocity components for $R_e = 1$. The velocity component u

on the vertical line x = 0.1 is shown in Figure 7. The present results in Figure 7 are in good agreement with the finite difference solution of Reference [16] using the vorticity stream function formulation. In Figure 7, it can be seen that the primitive variable solutions of Reference [19] show wiggles at points y = 0.7 and y = 0.8. The velocity component v for the same Reynolds number $R_e = 1$ is shown in Figure 8 on the horizontal centerline. The finite element methods predict higher values for the velocity component v than the finite difference solutions. The same trend is shown in Figure 9 for the velocity component u on the vertical centerline.

CONCLUSIONS

A method to obtain converged solutions for the Poisson equation with Neumann boundary conditions is presented. The computation time in the present method is approximately twice the computation time required for the solution of a Poisson equation with Dirichlet boundary conditions on the same domain. The present method extends the validity of the Direct Solvers to the solution of the Poisson equation with Neumann boundary conditions on general domains. The application of this method in the solution of Navier-Stokes equations makes the primitive variables formulation competitive with the vorticity stream function formulation.

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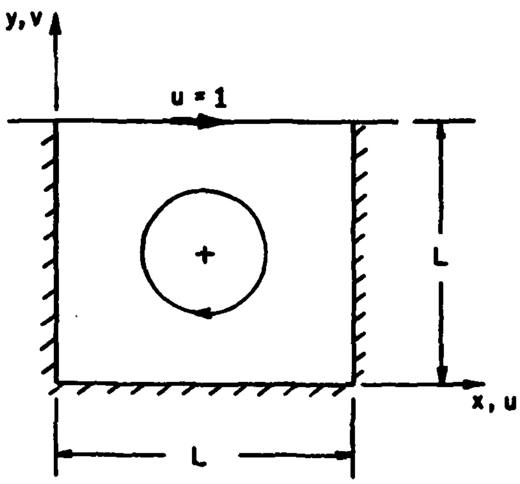


Figure 1. Cavity Geometry and Dimensions.

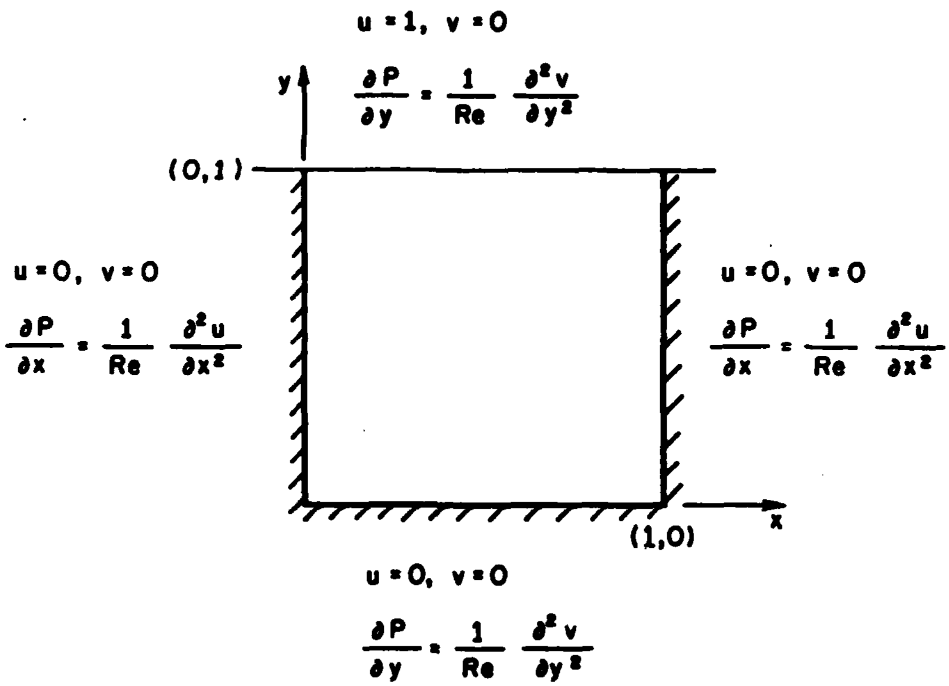


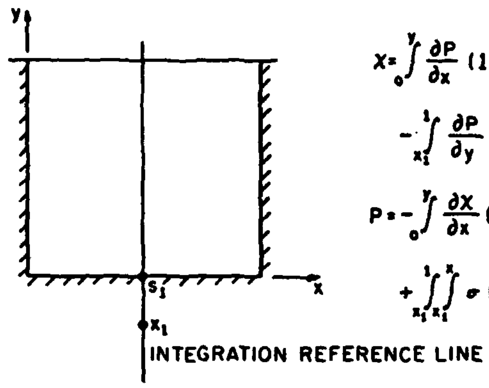
Figure 2. Boundary Conditions in Primitive Variables.

$$\chi = -\int_0^x \frac{\partial P}{\partial y}(x,1)\,dx - \int_0^1\int_{x_i}^0 \sigma\,dx\,dy + \int_0^1 \frac{\partial P}{\partial x}(0,y)\,dy - \int_{x_i}^0 \frac{\partial P}{\partial y}(x,0)\,dx$$

$$\frac{\partial P}{\partial y} = \frac{1}{Re}\;\frac{\partial^2 v}{\partial y^2}$$

$$\chi = \int_0^y \frac{\partial P}{\partial x}(0,y)\,dy - \int_0^y\int_{x_i}^0 \sigma\,dx\,dy - \int_{x_i}^0 \frac{\partial P}{\partial y}(x,0)\,dx$$

$$P = -\int_0^y \frac{\partial \chi}{\partial x}(0,y)\,dy + \int_{x_i}^0 \frac{\partial \chi}{\partial y}(x,0)\,dx + \int_{x_i}^0\int_{x_i}^x \sigma(x,0)\,dx\,dx$$



$$\chi = \int_0^y \frac{\partial P}{\partial x}(1,y)\,dy - \int_0^y\int_{x_i}^1 \sigma\,dx\,dy - \int_{x_i}^1 \frac{\partial P}{\partial y}(x,0)\,dx$$

$$P = -\int_0^y \frac{\partial \chi}{\partial x}(1,y)\,dy + \int_{x_i}^1 \frac{\partial \chi}{\partial y}(x,0)\,dx + \int_{x_i}^1\int_{x_i}^x \sigma(x,0)\,dx\,dx$$

$$\chi = -\int_{x_i}^x \frac{\partial P}{\partial y}(x,0)\,dx$$

$$P = \int_{x_i}^x \frac{\partial \chi}{\partial y}(x,0)\,dx + \int_{x_i}^x\int_{x_i}^x \sigma(x,0)\,dx\,dx$$

Figure 3. Dirichlet Boundary Conditions for χ and P.

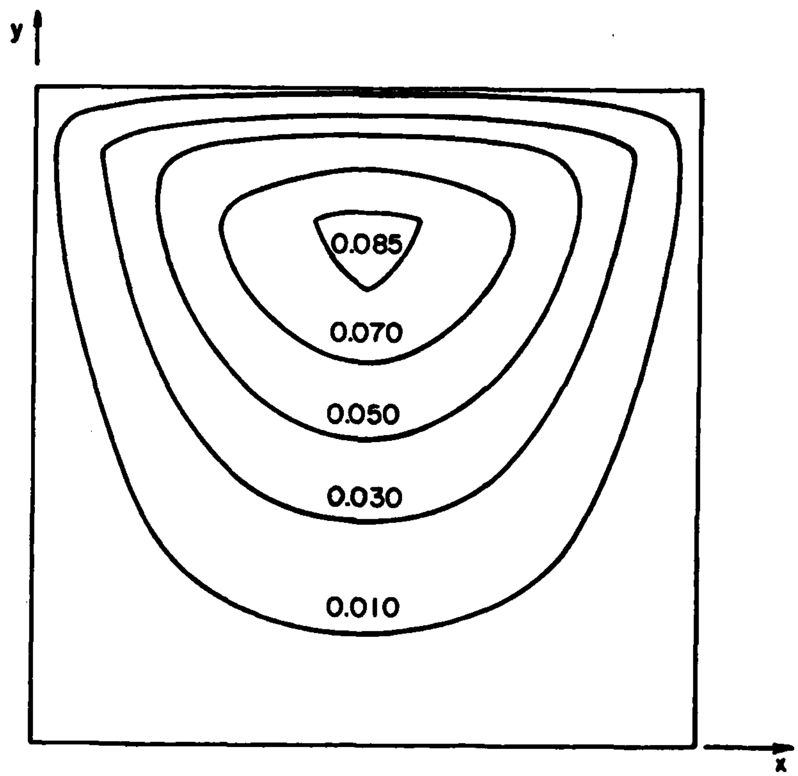


Figure 4. Stream Function Contours.

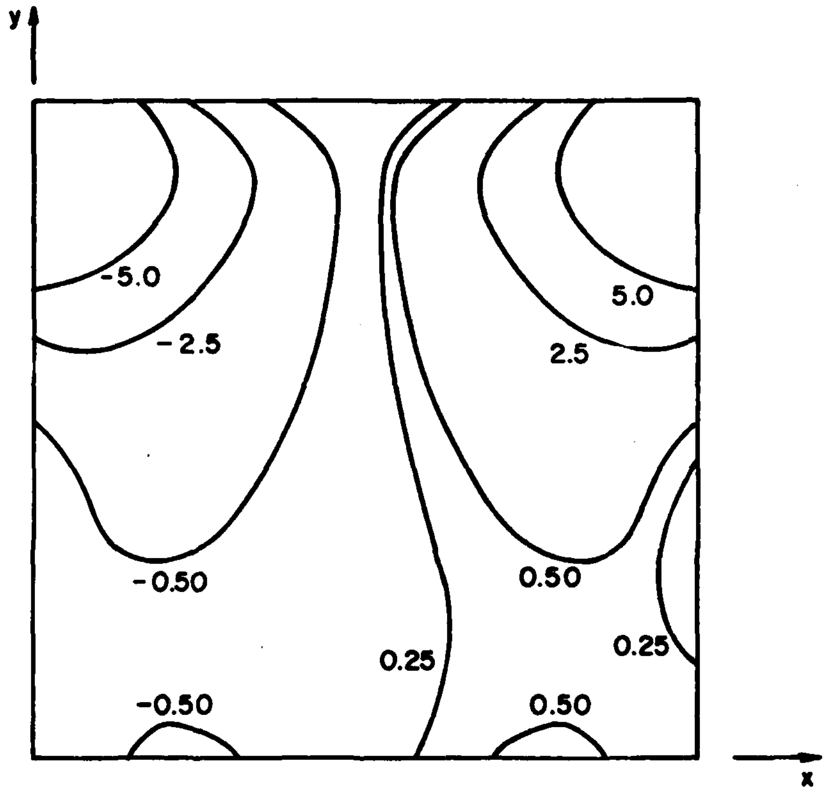


Figure 5. Static Pressure Contours.

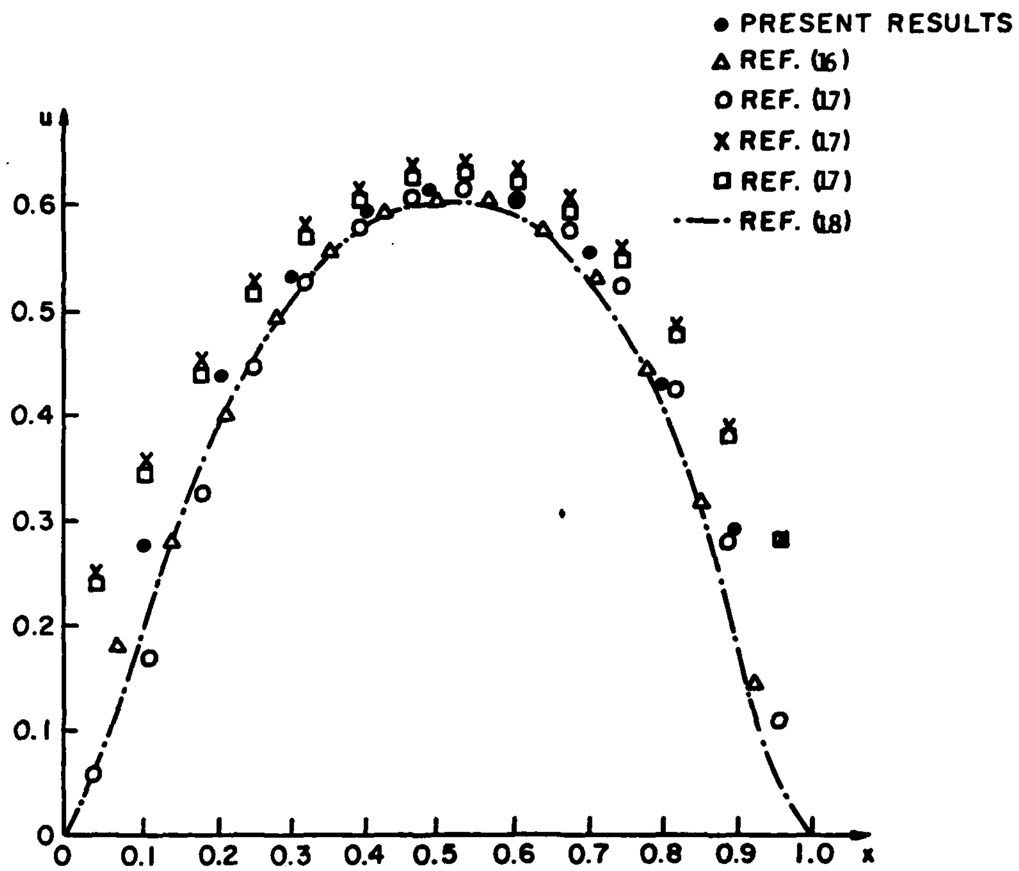


Figure 6. Velocity Component u at y = 1/14 from moving wall, Re = 10.

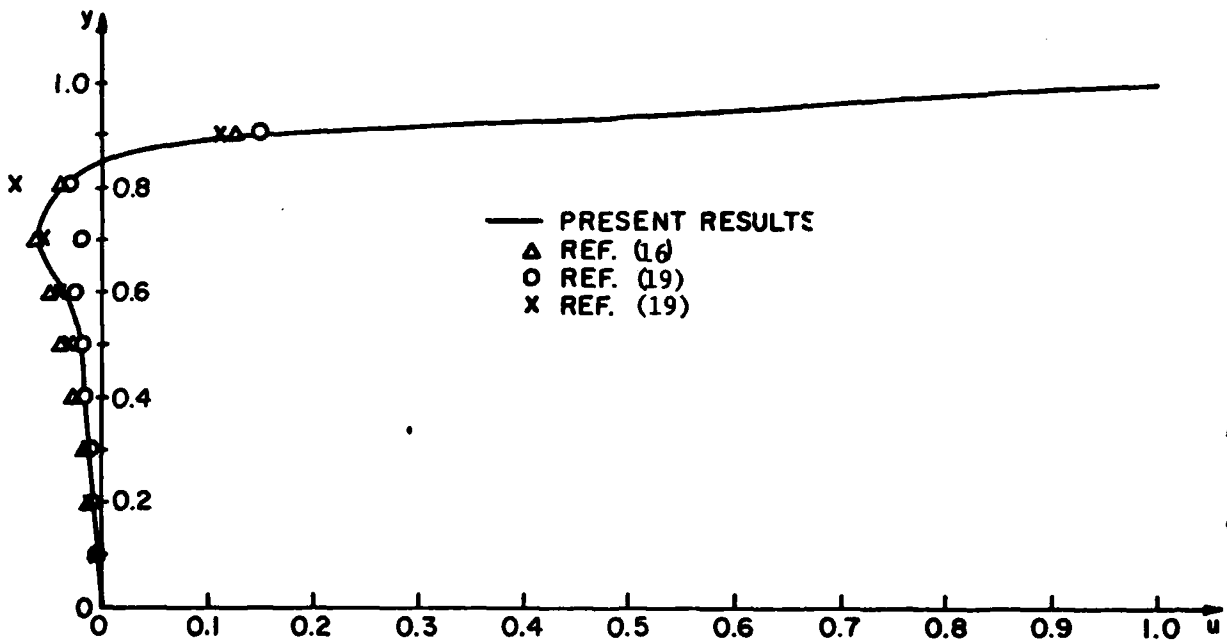


Figure 7. Velocity Component u at x = 0.1, Re = 1.

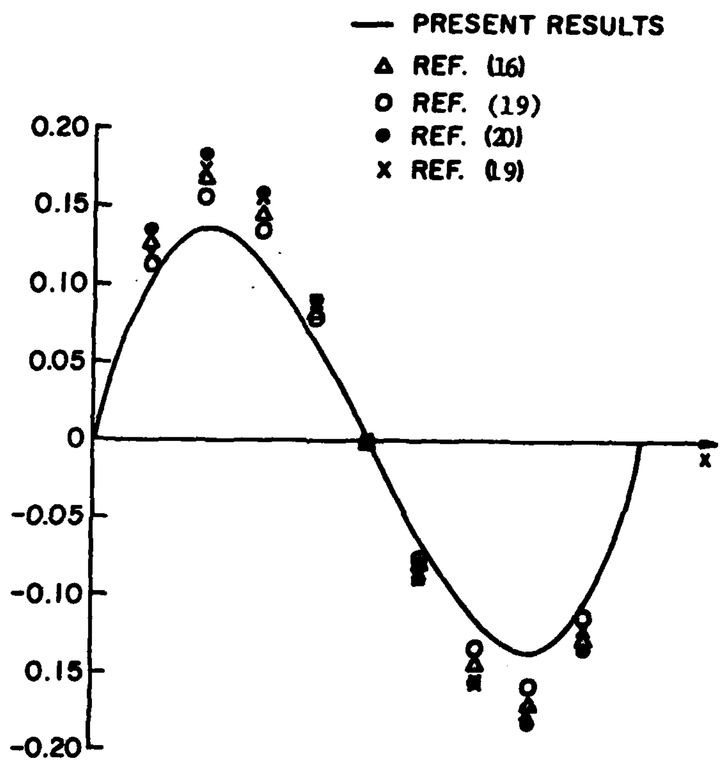


Figure 8. Velocity Component v at y = 1/2, Re = 1.

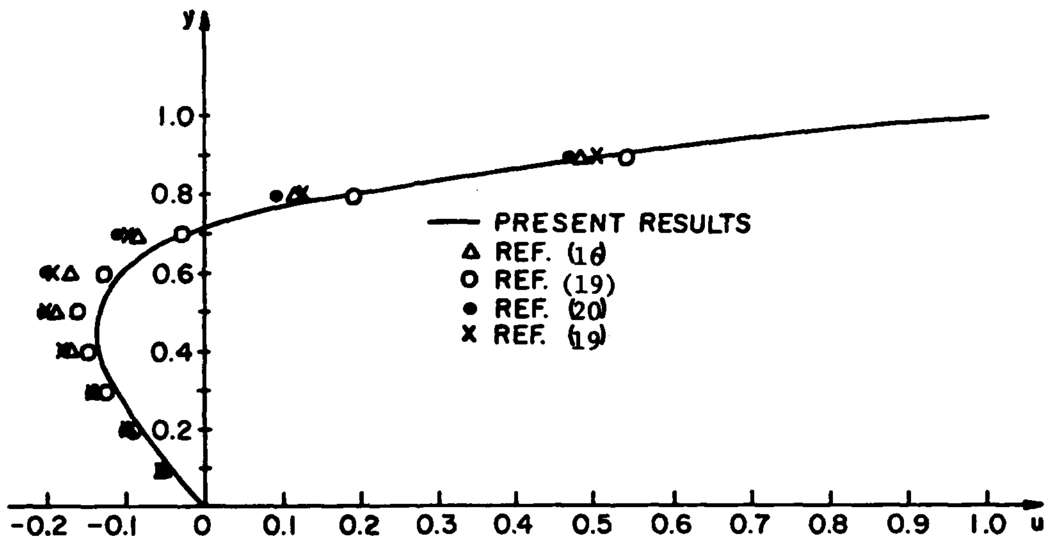


Figure 9. Velocity Component u at x = 1/2, Re = 1.

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